Superluminal Group Velocity of Electromagnetic Near-fields *

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Superluminal phenomena have been reported in many experiments of electromagnetic wave propagation, where the superluminal behaviors of evanescent waves are the most interesting ones with the genuine physical significances. Consider that evanescent waves are related to the near-zone fields of electromagnetic sources, based on the first principles, we study the instantaneous group velocities of electromagnetic fields in near-field region, and show that they can be superluminal, which can provide a heuristic understanding for the superluminal properties of evanescent waves.

Keywords: near fields, evanescent waves, group velocity, superluminal

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1. Introduction

A series of recent experiments have revealed that electromagnetic wave was able to travel at a group velocity faster than the velocity of light in vacuum, or at a negative group velocity. For example, these phenomena have been observed in dispersive media [1-4], in electronic circuits [5], and in evanescent wave cases [6-13]. In fact, over the last decade, the discussion of the tunneling time problem has experienced a new stimulus by the results of analogous experiments with evanescent electromagnetic wave packets [14-19], and the superluminal effects of evanescent waves have been revealed in photonic tunneling experiments in both the optical domain and the microwave range [6-13]. All the experimental results have shown that the phase time do describe the barrier traversal time [13,20].

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In view of the fact that the evanescent waves are actually attributed to the near-zone fields of electromagnetic sources, in this article, we shall show that the group velocities of electromagnetic near-fields can be superluminal.

2. General expressions of phase velocity and group velocity

There has several different methods to introduce the concepts of phase velocity and group velocity, here we will choose a most general method. In the most general case, one can expand a field quantity $\Phi(\boldsymbol{r},t)$ as the superposition of different frequency components with the *usual* Fourier-transform as its special case ($\boldsymbol{r}=(x,y,z)$):

$$\Phi(\boldsymbol{r},t)=\frac{1}{(2\pi)^{3/2}}\int_{-\infty}^{+\infty}\Psi(\boldsymbol{r},t,\boldsymbol{k},\omega)\exp[\mathrm{i}\Theta(\boldsymbol{r},t,\boldsymbol{k},\omega)]\mathrm{d}^3k \qquad (1)$$

where $\mathrm{i}=\sqrt{-1}$, $\mathrm{d}^3k=\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}k_z$, $\omega=\omega(k)$ is the frequency, $\boldsymbol{k}=(k_x,k_y,k_z)$ the wave-number vector, and $\Psi(\boldsymbol{r},t,\boldsymbol{k},\omega)$ the amplitude of the *k*-th component of $\Phi(\boldsymbol{r},t)$. Let $r=|\boldsymbol{r}|$, $k=|\boldsymbol{k}|$ and so on. Only expand $\Phi(\boldsymbol{r},t)$ as the superpositions of monochromatic plane waves can one write $\Phi(\boldsymbol{r},t)$ as the *usual* Fourier-transform form. While, if we expand $\Phi(\boldsymbol{r},t)$ as the superposition of monochromatic *cylindrical (or spherical, etc.)* waves, the amplitude $\Psi(\boldsymbol{r},t,\boldsymbol{k},\omega)$ would depend on space (or even time) coordinates. However, the validity of our later conclusions has nothing to do with the fact whether Equ. (1) corresponds the *usual* Fourier-transform or not. The quantity $\Theta(\boldsymbol{r},t,\boldsymbol{k},\omega)$ in Equ. (1) represents a phase. Following the definition of phase velocity, we let $\Theta(\boldsymbol{r},t,\boldsymbol{k},\omega)=C$ (C is a constant) and have $\mathrm{d}\Theta/\mathrm{d}t=(\partial\Theta/\partial r)(\mathrm{d}r/\mathrm{d}t)+\partial\Theta/\partial t=0$, then the phase velocity is ($\boldsymbol{a}_r=\boldsymbol{r}/r$)

$$\boldsymbol{v}_{\mathrm{p}}=\boldsymbol{a}_r\frac{\mathrm{d}r}{\mathrm{d}t}=-\boldsymbol{a}_r(\frac{\partial\Theta}{\partial t})/(\frac{\partial\Theta}{\partial r}) \qquad (2)$$

On the other hand, we may take group velocity (say, $\boldsymbol{v}_{\mathrm{g}}$) as the move velocity of the peak of wave packet, which is valid for both deformed and undeformed wave packets, and in agreement with the phase time theory of tunneling time (note that the phase time has nothing to do with the concept of phase velocity) [15-17]. In fact, in quantum mechanics, the classical velocity of a particle corresponds to the group velocity of a wave packet that represents the particle, in spite of the fact that the wave packet is deformed with time (provided that the particle has nonzero mass). According to Ref. [21], the group velocity may be meaningful even for broad band pulses and when the group velocity is superluminal or negative. Using Equ. (1), and consider that the superposition of different frequency components gives an extremum at peak location (say, r_c) of the wave packet $\Phi(\boldsymbol{r},t)$, namely, in the stationary-phase approximation, the peak location r_c is given by $\partial\Theta/\partial k=0$, accordingly the group velocity is $\boldsymbol{v}_g=\boldsymbol{a}_r\,\mathrm{d}r_c/\mathrm{d}t$. Let $\Delta\equiv\partial\Theta/\partial k=0$, then we have $\mathrm{d}\Delta/\mathrm{d}t=0$, that is, $(\partial\Delta/\partial r)(\mathrm{d}r/\mathrm{d}t)+\partial\Delta/\partial t=0$, one obtains

$$\boldsymbol{v}_g=-\boldsymbol{a}_r\,(\frac{\partial\Delta}{\partial t})\Big/(\frac{\partial\Delta}{\partial r})=-\boldsymbol{a}_r\,(\frac{\partial^2\Theta}{\partial t\partial k})\Big/(\frac{\partial^2\Theta}{\partial r\partial k}) \tag{3}$$

Equ. (2) and Equ. (3) are the general expressions for calculating the phase velocity and group velocity, respectively. As an example, let $\Theta=\omega t-\boldsymbol{k}\cdot\boldsymbol{r}$, one has $\boldsymbol{v}_p=\boldsymbol{a}_r\,\omega/k$ and $\boldsymbol{v}_g=\boldsymbol{a}_r\,\partial\omega/\partial k$.

Furthermore, let us consider that the phase and group velocities along a given direction, say, the z-axis direction. Let k_z denotes the projection of $\boldsymbol{k}$ in the z-axis direction, it is easy to show that the phase and group velocities along the z-axis direction are, respectively

$$\boldsymbol{v}_{pz}=-\boldsymbol{a}_z\,(\frac{\partial\Theta}{\partial t})\Big/(\frac{\partial\Theta}{\partial z}),\quad \boldsymbol{v}_{gz}=-\boldsymbol{a}_z\,(\frac{\partial^2\Theta}{\partial t\partial k_z})\Big/(\frac{\partial^2\Theta}{\partial z\partial k_z}) \tag{4}$$

where $\boldsymbol{a}_z$ is the unit vector along the z-axis direction. Equ. (4) are the most general expressions for calculating the phase and group velocities.

It is highly important to note that, even though in the absence of dispersion, one has $\boldsymbol{v}_{pz}\neq\boldsymbol{v}_{gz}$ provided that $|k_z|\neq|\boldsymbol{k}|$.

3. Group velocities of the near-zone fields of antennas

To gain physics insight, we restrict ourselves to the fields of an electric dipole antenna without loss of generality. For a system of charges and currents varying in time we can make a Fourier analysis of the time dependence and handle each Fourier component separately [22]. We therefore lose no generality by considering the fields of an electric dipole antenna with currents varying sinusoidally in time:

$$\boldsymbol{J}(\boldsymbol{x},t)=\boldsymbol{J}(\boldsymbol{x})\exp(\mathrm{i}\omega t) \tag{5}$$

As usual, the real part of such expressions is to be taken to obtain physical quantities. In the Lorentz gauge, the corresponding vector potential $\boldsymbol{A}(\boldsymbol{x},t)$ is

$$\boldsymbol{A}(\boldsymbol{x},t)=\frac{\mu}{4\pi}\int \boldsymbol{J}(\boldsymbol{x}')\frac{\exp\mathrm{i}(\omega t-\boldsymbol{k}\cdot\boldsymbol{r})}{r}\mathrm{d}^3x' \quad (6)$$

Where $r=|\boldsymbol{r}|=|\boldsymbol{x}-\boldsymbol{x}'|$, $k=|\boldsymbol{k}|$, $\boldsymbol{k}$ is the wave-number vector, μ is the vacuum permeability. Furthermore, we assume that the electric dipole antenna, as a linear antenna, is oriented along the z axis, extending form $z=-d/2$ to $z=d/2$, and its length d satisfies $d\ll\lambda$, where $\lambda=2\pi c/\omega$ is the wavelength. The magnetic induction is $\boldsymbol{B}=\nabla\times\boldsymbol{A}$ while, outside the source, the electric field is $\boldsymbol{E}=\mathrm{i}\nabla\times\boldsymbol{B}/k$.

On the basis of Ref. [22-23], in spherical coordinate system (r,θ,φ), $\boldsymbol{k}\cdot\boldsymbol{r}=kr$, and the non-zero components of the electromagnetic field of the antenna are:

$$\begin{aligned}
H_\varphi &= H_0\frac{\sin\theta}{r}(1+\frac{1}{\mathrm{i}kr})\exp[\mathrm{i}(\omega t-kr)]\\
E_r &= E_1\frac{\cos\theta}{r^2}(1+\frac{1}{\mathrm{i}kr})\exp[\mathrm{i}(\omega t-kr)]\\
E_\theta &= E_2\frac{\sin\theta}{r}(1+\frac{1}{\mathrm{i}kr}-\frac{1}{k^2r^2})\exp[\mathrm{i}(\omega t-kr)]
\end{aligned} \quad (7)$$

Where θ is the angle between the direction of observation (along $\boldsymbol{r}$) and the polarization direction of the electric dipole moment, H_0, E_1, E_2 are purely real or purely imaginary constants. In the near zone, $0<kr\ll 1$, such that $H_\varphi/E_r\approx 0$ and $H_\varphi/E_\theta\approx 0$, thus the non-zero components of the electromagnetic field of the antenna can be written as

$$\begin{aligned}
E_r &= E_1\frac{\cos\theta}{r^2}(1+\frac{1}{\mathrm{i}kr})\exp[\mathrm{i}(\omega t-kr)]\equiv|E_r|\exp(\mathrm{i}\Theta+C_1)\\
E_\theta &= E_2\frac{\sin\theta}{\mathrm{i}kr^2}(1+\frac{1}{\mathrm{i}kr})\exp[\mathrm{i}(\omega t-kr)]\equiv|E_\theta|\exp(\mathrm{i}\Theta+C_2)
\end{aligned} \quad (8)$$

where C_1 and C_2 are two constants, and the phase factor Θ is:

$$\Theta=\omega t-kr+\arctan(-1/kr) \quad (9)$$

where $\arctan(x)$ is the inverse tangent function of x. Using Equs. (2)-(3) and $\boldsymbol{a}_r=\boldsymbol{r}/r$, we obtain the phase velocity $\boldsymbol{v}_\mathrm{p}$ and group velocity $\boldsymbol{v}_\mathrm{g}$ as follows:

$$\boldsymbol{v}_{\mathrm{p}} = \boldsymbol{a}_r [1 + \frac{1}{(kr)^2}] c = \boldsymbol{a}_r v_{\mathrm{p}} \qquad (10)$$

$$\boldsymbol{v}_{\mathrm{g}} = \boldsymbol{a}_r [\frac{(kr)^4 + 2(kr)^2 + 1}{(kr)^4 + 3(kr)^2}] c = \boldsymbol{a}_r v_{\mathrm{g}} \qquad (11)$$

What we are interested in is only the group velocity. We give the diagrammatic curve of v_{g}/c (instead of v_{g}) *vs* kr.

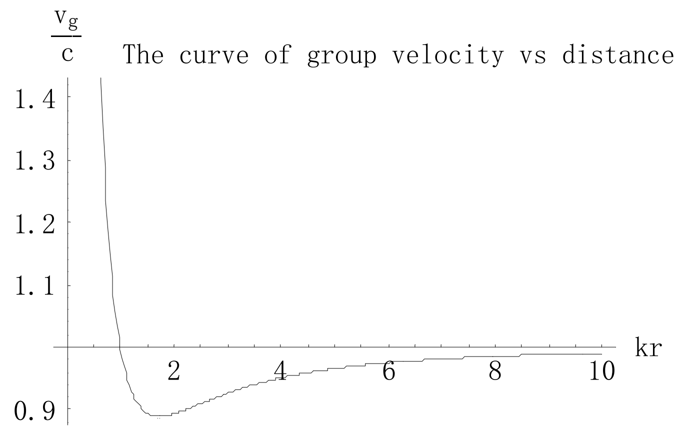

Fig. 1 The curve of group velocity vs distance

Based on Fig. 1, we summarize the main characters of the group velocities of the near fields as follows:

1) As $kr > 1$, the group velocities of the near fields satisfy $8c/9 \approx 0.9c < v_{\mathrm{g}} < c$, and, as $kr \to +\infty$ one has $v_{\mathrm{g}} \to c$. Therefore, in the far zone, the group velocities of the near fields are subluminal;

2) As $0 < kr \le 1$, in contrast to the case 1), the group velocities of the near fields are greater than or equal to the velocity of light in vacuum. Therefore, in the near zone, the group velocities of the near fields are superluminal.

4. Near-zone fields and evanescent waves

As an example, we show that there exists a close similarity between the evanescent waves inside an undersize waveguide and the near-zone fields of antennas:

a) Distinct from the travelling waves inside an ordinary waveguide, the evanescent waves inside an undersize waveguide, attenuate exponentially along the direction of propagation, and their average power flows do not exist (but exist as an energy storage), which due to the fact that the impedance is purely capacitive (for the TM mode) or inductive (for the TE mode). As for the energy storage in the undersize waveguide, the electric energy is more than magnetic energy for the TM mode and on the contrary for the TE mode.

b) Similarly, the near-zone fields of an antenna are also sharply attenuated as compared with the far-zone fields of the antenna, and the average power flows of the near-zone fields do not exist because the impedance is purely capacitive (for the electric dipole antenna) or inductive (for the magnetic dipole antenna), it is only an energy storage that exists. As for the energy storage of the near-zone fields, the electric energy is more than magnetic energy for the electric dipole antenna and on the contrary for the magnetic dipole antenna.

In fact, in Ref. [24], Feynman has given a detailed analysis for the equivalence between the evanescent waves inside a waveguide and the near fields of a source

5. Conclusions and prospects

On the one hand, the superluminal behaviors of evanescent waves have been revealed in photonic tunneling experiments. On the other hand, in this article we have shown that the group velocity of the near-zone fields is superluminal. Owing to the fact that evanescent waves actually correspond to the near-zone fields of a source, the results obtained in this article can provide a heuristic understanding for the superluminal phenomena of the evanescent waves. In fact, according to quantum mechanics, the position-momentum uncertainty would become remarkable as a measurement is performed in the near zone of a source, such that the superluminal behaviors of the near-zone fields take place without violating causality principle.

In our next work, we shall provide a more quantitative analysis for the relations between the superluminal behaviors of the near-zone fields and those of the evanescent waves, and propose that these superluminal phenomena, without violating causality principle, have a common theoretical foundation in quantum field theory.

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